Chapter 8: Further Applications of Integration

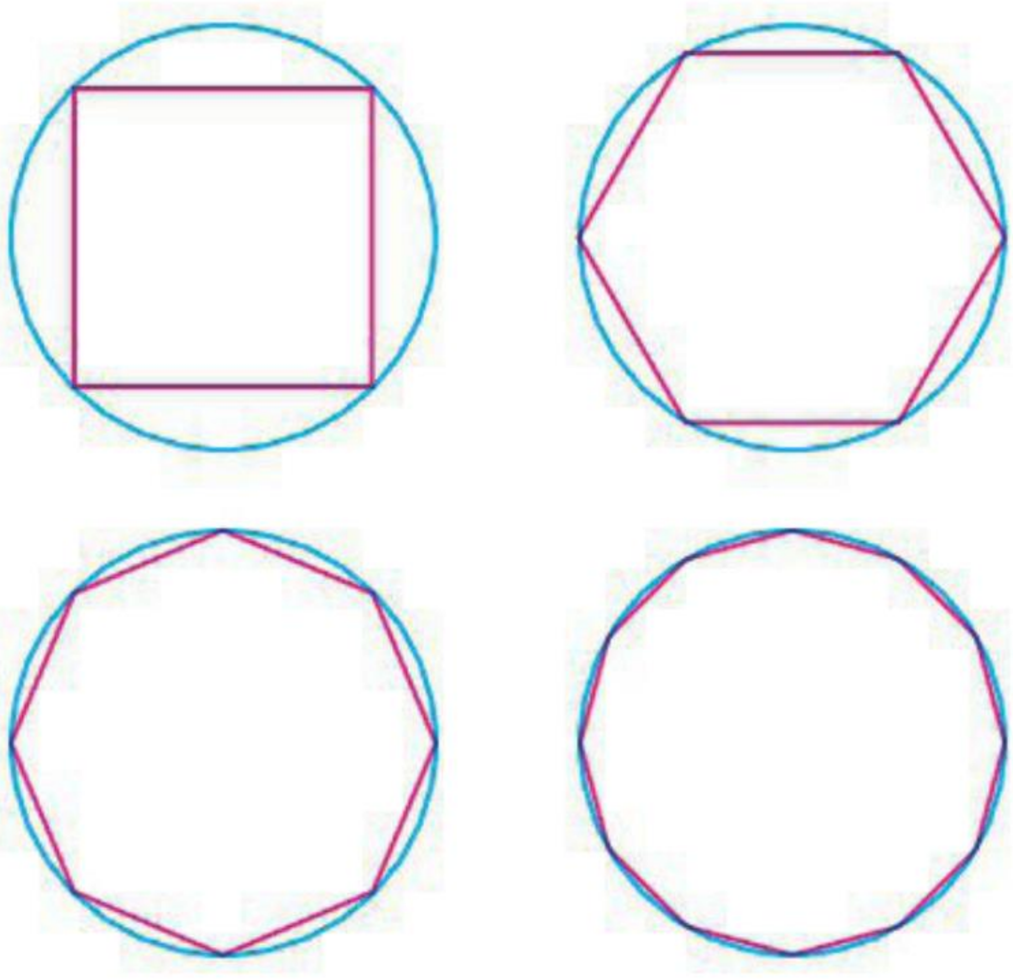
Section 8.1: Arc Length

Section 8.2: Area of a Surface of Revolution

Section 8.1: Arc Length

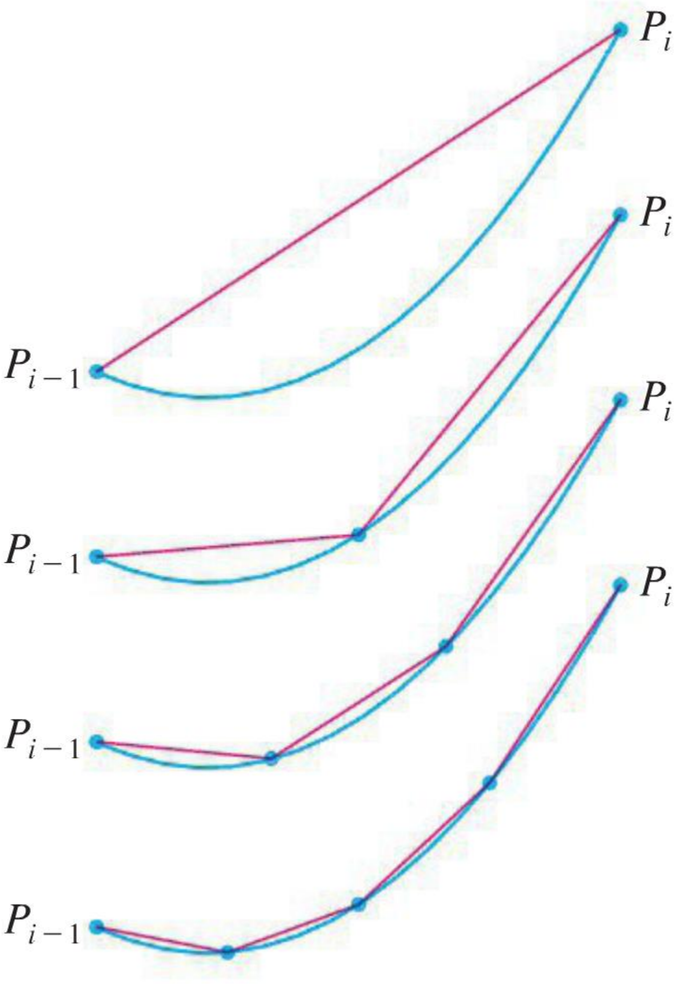
Arc Length

Derivation of Arc Length Formula…



Arc Length

Derivation of Arc Length Formula…



Arc Length

Derivation of Arc Length Formula…

y as a function of x version…

2 **The Arc Length Formula** If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leqslant x \leqslant b$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \qquad \text{or} \qquad L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Arc Length

Derivation of Arc Length Formula…

x as a function of y version…

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$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Arc Length

Ex 1: Find the exact length of the curve $y = 1 + 6x^{3/2}$ on $0 \leq x \leq 1$.

Arc Length

Ex 2: Find the exact length of the curve

$x = \frac{y^4}{8} + \frac{1}{4y^2}$ on $1 \leq y \leq 2$.

Arc Length

Ex 3: Find the length of the arc of the parabola $y^2 = x$ from $(0,0)$ to $(1,1)$.

Arc Length

Ex 4: Use Simpson's Rule with $n = 10$ to estimate the arc length of the curve $y = x\sin(x)$ on $0 \leq x \leq 2\pi$.

The Arc Length Function

Discussion…

If C is a smooth curve with equation $y = f(x)$ on $a \le x \le b$, and $s(x)$ is the length of the curve C from the initial point $P_0(a, f(a))$ to the point $Q\big(x, f(x)\big)$, then $s(x)$ is a function called the <u>arc length</u> function given by…

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$$

The Arc Length Function

Discussion…

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$$

The Arc Length Function

Ex 5: Find the arc length function for the curve $y = sin^{-1}(x) + \sqrt{1 - x^2}$ with starting point (0,1).

The Arc Length Function

Ex 6: "Find" the length of the arc of the curve $x^2 = (y-4)^3$ from $P(1,5)$ to $Q(8,8)$.

Section 8.2: Area of a Surface of Revolution

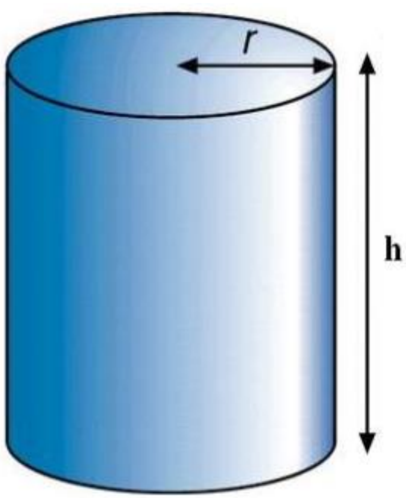
Area of a Surface of Revolution

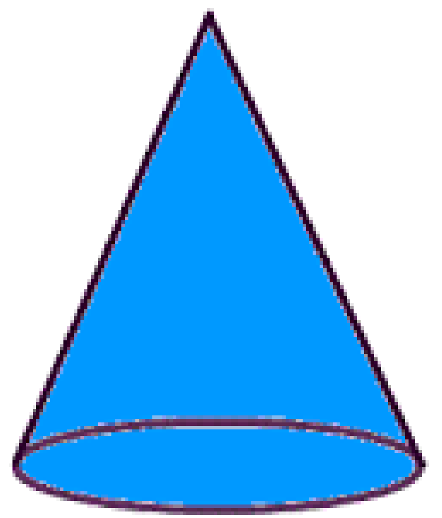
What does surface area mean?

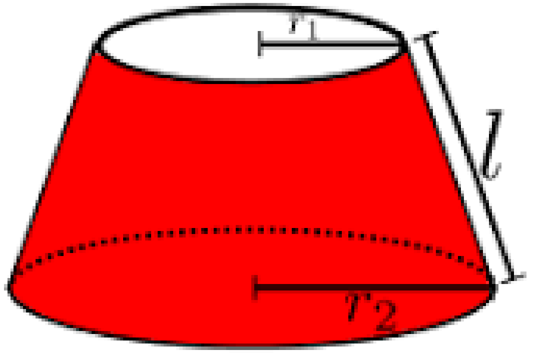
We will actually be finding the lateral surface area (ignoring the area of the ends of the object)

Find the surface are of a…

Right circular cylinder cone frustum of a cone







Area of a Surface of Revolution

Definition of (lateral) surface area of revolution:
Let $f(x)$ be a function defined on [a, b] whose graph is always on or above the x-axis. Take the region (area) between the curve and the x-axis from a to b and rotate it around the x-axis to generate a volume.
We will actually be finding the lateral surface area (ignoring the area of the ends of the object)

Find the surface area of a…

Right Circular Cylinder	Cone	Frustum of a Cone
$2\pi rh$	πrl	$2\pi rl$ where $r = \frac{r_1+r_2}{2}$

Area of a Surface of Revolution

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$

$$S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_c^d 2\pi y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = \int 2\pi y\,ds$$

$$S = \int 2\pi x\,ds$$

Area of a Surface of Revolution

Ex 1: The curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x-axis.

Area of a Surface of Revolution

<u>Ex 2</u>: The arc of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$ is rotated about the y-axis. Find the area of the resulting surface.

Area of a Surface of Revolution

Ex 3: Find the area of the surface generated by rotating the curve $y = e^x$, $0 \le x \le 1$, about the x-axis.